

DEVELOPING A METACOGNITIVE TEACHING MODEL FOR PROBLEM SOLVING IN MIDDLE SCHOOL MATHEMATICS

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Abstract

The main purpose of this study is to investigate the effectiveness of the metacognitive teaching model for teaching mathematics at the middle school level in Myanmar. In this study, a mix-methods (QUAN → qual) design was adopted. Quasi-Experimental design was adopted to collect the quantitative data and case study design was applied to collect the qualitative data. It was started in the first week of November 2021 and ended in the second week of January 2022. The study was geographically restricted to Yangon Region and a total of (258) Grade six students participated. A pretest, a posttest, a metacognitive skills inventory questionnaire, and an observation checklist were used as the research instruments. The pretest and posttest data were analyzed through one-way ANCOVA and it was found that there were significant differences in the mathematics achievement on posttest between the experimental and control groups in all selected schools. Students' responses to the metacognitive skills inventory questionnaire were analyzed through Wilcoxon Signed Rank Test and the results showed that there were significant differences in metacognitive skills of experimental group students before and after the treatment. Students' problem solving behaviors were analyzed through the think-aloud protocol analysis method and it was found that almost all students exhibited both cognitive and metacognitive behavior. Additionally, qualitative findings supported the quantitative findings. Therefore, the research findings proved that the proposed metacognitive teaching model has a positive contribution to teaching problem solving at the middle school level in Myanmar.

Keywords: Metacognition, Teaching Model, Problem Solving, Achievement in Mathematics, Metacognitive Skills.

Introduction

In today's world, the exponential growth of technology expects such kinds of individuals who can apply mathematical knowledge to solve ill-defined problems. Since the 1980s, the studies conducted on mathematical problem solving have emphasized if students are capable of monitoring their thinking process while solving problems and the term metacognition has been recognized as a key factor in problem solving (Ken, Clements, & Ellerton, 1996).

Purposes of the Study

The main purpose of this study is to investigate the effectiveness of the metacognitive teaching model for teaching mathematics at the middle school level in Myanmar.

The specific objectives of the study are as follows:

1. To study the effects of the metacognitive teaching model on the achievement of students in mathematics.
2. To investigate the effects of the metacognitive teaching model on the metacognitive skills of students concerning mathematical problem solving.
3. To explore the problem solving behaviors of students while solving mathematics problems.

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Research Questions

1. Is there a significant difference in the achievement of mathematics between students who receive the metacognitive teaching and those who do not receive it?
2. Is there a significant difference in the metacognitive skills of the experimental groups concerning mathematical problem solving before and after the treatment?
3. Which behaviors do students exhibit while solving mathematics problems?

Scope of the Study

This study is geographically limited to Yangon Region. Participants in this study are Grade six students from the selected schools in (2021-2022) Academic Year. The duration of the treatment period is limited to eight weeks timeframe. The lesson contents are limited to six content areas prescribed in Grade six mathematics textbook volume I and II.

Definition of Key Terms

Metacognition. Metacognition refers to thinking about thinking, and its main function is to plan, direct, control, examine, and evaluate all cognitive thinking processes; covering critical and creative thinking; to make appropriate decisions to solve a problem (Sang, 2003).

Teaching model. A teaching model is an overall plan or pattern to learn specific kinds of knowledge, attitudes, and skills. It has a theoretical basis or philosophy behind it and encompasses specific teaching steps designed to accomplish desired educational outcomes (Joyce & Weil, 1972, cited in Arends, 2007).

Problem solving. Problem solving is a multiple steps process where the problem solver must find relationships between the past experience (schema) and the problem at hand and then guide thinking directed towards the successful solution of a problem (Mayer, 1980).

Achievement in mathematics. Achievement in mathematics refers to the student's scores on the posttest which is developed based on the five process skills of mathematics described by the National Council of Teachers of Mathematics (NCTM) in April 2000.

Metacognitive skills. Metacognitive skills refer to concrete metacognitive activities that occur at the onset of task performance (orientation), during task performance (planning, monitoring, evaluation), and at the end of task performance (reflection and elaboration) (Steal, Veenman, Deelen, & Haenen, 2010).

Statement of the Problem

One of the problems encountered by Myanmar students while doing problem solving is that they focus on getting the answer and if the answer is right, they do not check, evaluate, and reflect on the whole process and move to the next problem. Sometimes, if the answer is right by chance and the steps of the solution are wrong, that will lead to underachievement. Another problem stated by Hardman, Stoff, Aung, and Elliott (2014) is the guided co-construction of knowledge in which a teacher guides students' cognition and creates opportunities for collaborative learning to promote critical thinking and problem solving skills is rarely observed in mathematics classrooms.

Significance of the Study

Research findings point out that teaching problem solving through metacognition promotes students to be self-directed learners who are self-reliant to seek the solution to any kind of problem (Zimmerman, 2000). Metacognition and problem solving are interconnected and teaching problem solving through metacognition results in better achievement in problem solving. However, there is

no practice in Myanmar to confirm the theory and advocate the previous research findings. Therefore, mathematics education in Myanmar needs a new contribution that considers metacognition such as planning how to approach a given learning task, monitoring the progress, evaluating the result, and reflecting on the completion of the whole task for developing students' achievement and metacognitive skills in mathematical problem solving.

Review of Related Literature

Philosophical (Theoretical) Considerations

Progressivism places more emphasis on the process of learning than on the result. Mathematical problem solving involves reflection in action and reflection on action and these two concepts are closely related to metacognition (Schon, 1983). Cognitivism views that metacognition and problem solving are interrelated and these are the complex higher-order thinking in the human learning process. Metacognitive knowledge about problems and strategies, and the skills of planning, monitoring, and evaluation are essential for successful problem solving (Gredler, 2001). Constructivism also views learners as self-regulated and active participants in their learning (Churchill et al., 2013). Self-regulation also requires metacognitive mediators such as planning, monitoring, and evaluating (Schunk, 2012).

Psychological Considerations

Gestalt theorists pointed out that the process of problem solving is based on the whole and the part relationship. They described that mathematics teachers should encourage both reproductive and productive thinking in the mathematics classroom with the caution of giving ready-made steps (Katona, 1940, cited in Moslehpour, 1995). Bruner (1964, cited in Gredler, 2001) said that mathematical problem solving will be more effective and simpler by using symbols to represent abstract concepts and think-aloud strategy can be used effectively in teaching students what and how to think about mathematical problem solving.

According to Piaget's developmental stages, Grade six students fall into the formal operation stage and metacognition begins to develop during this stage and further in life (Flavell, 1977, cited in Tarricone, 2011). Besides, Vygotsky (1978) pointed out that students should be able to control their cognition with the help of inner speech, through the process of internalization. To reach this stage, they have to experience first the stage of egocentric speech. Additionally, Bandura (1977, cited in Gredler, 2001) also stated that cognitive modeling is one of the best strategies for demonstrating how to regulate cognition in mathematical problem solving and fosters the development of metacognition.

Conceptualization of Metacognition

The term metacognition appeared around 1975 in the work of cognitive psychologist John Flavell. Metacognition is a form of cognition, a second-order thinking process that involves active control over cognitive processes (Devine, 1993). Flavell (1976, cited in Desoete, 2008) described three major facets of metacognition, namely, metacognitive knowledge, metacognitive experience, and metacognitive skills. Metacognitive knowledge is knowledge, awareness, and a deeper understanding of one's cognitive processes and products (Flavell, 1976, cited in Desoete, 2008). Metacognitive experience is the conscious reactions and self-judgments regarding personal performance before, during, and after task execution (Rosenzweig, Krawec, & Montague, 2011). Metacognitive skills refer to the authentic procedures and strategies used during task execution to monitor and control one's cognition (Rosenzweig et al., 2011). Four important metacognitive skills for mathematical problem solving are (i) prediction, (ii) planning, (iii) monitoring, and (iv) evaluation skills (Desoete, 2008).

Prediction (orientation) is the skill that enables students to think about the learning objectives, proper learning characteristics, available time, and difficulty of a task. Planning skill makes students think in advance about how, when, and why to obtain their purpose through a sequence of sub-goals leading to the main goal of the problem. In mathematical problem solving, planning involves analyzing the problems, retrieving relevant domain-specific knowledge and skills, exploring the strategies, and arranging the problem solving steps. Monitoring is the self-regulated control of the thinking process during the solution process. Evaluation skill can be defined as the reflection that takes place after an event has finished (Brown, 1987, cited in Desoete, 2008). Evaluation skills enable students to assess their performance, compare their task performance with others, and discover errors within the problem solving process.

Metacognitive knowledge and experiences are constantly interrelated and metacognitive skills control and monitor the cognitive processes. Thus, mathematical problem solving should be taught through metacognition to facilitate students' learning and develop thinking skills.

Theoretical Perspectives on Mathematical Problem Solving

Metacognitive skills in problem solving refer to the knowledge and processes used to guide the thinking directed towards the successful solution of the problem (McCormic, 2003). Metacognitive skills help students to define the problem, select an appropriate solution strategy, monitor the effectiveness of the solution strategy, and identify and overcome obstacles to solving the problem (Davidson & Sternberg, 1998, cited in McCormic, 2003).

According to Mayer (1998), successful problem solving depends on three components: (i) skills, (ii) meta-skills, and (iii) will. In these three components, metacognition in the form of meta-skill is central in problem solving because it monitors and controls the other components. Wilson, Fernandez, and Hadway (1993) described that problem solving involves exploration, pattern finding, and mathematical thinking with consideration about teaching how to think as opposed to what to think or what to do.

Problem solving in mathematics is helpful in the proper development of one's mental power. No matter what types of problems are submitted, students who are effective problem solvers identify the problem, plan the strategy, ask themselves whether they are doing makes sense or not, adjust their problem solving strategies when necessary, and look back to reflect on the reasonableness of their solution and their approaches.

Components of the Proposed Metacognitive Teaching Model

The proposed model is based on theoretical concepts of the information processing model, basic teaching model, psychological cybernetic model, algorithmic model, heuristic/plan generating model, multiplicity model, and multi-modal model. Additionally, the components of Brown's model, general problem solving model, Polya's problem solving model, and IDEAL problem solving model were taken into consideration.

The proposed model is composed of six components (see Figure1). The explanations of each component are presented as follows.

(i) Stimulating/Eliciting domain specific knowledge. Domain specific knowledge is information that leads action to complete specific tasks. Thus, task-relevant prior knowledge to the student is elicited at the beginning of the lesson.

(ii) Informing learning outcomes. In the second stage, learning outcomes are informed to students to provide a set of shared expectations between the teacher and the students.

(iii) Presenting the problem. In the third stage, a word problem from the prescribed textbook is presented to all students. This stage includes reading aloud, silent reading, and verbalization of the parts of the problem statement.

(iv) Solving the problem through explicit modeling/think-aloud. This component is based on social cognitive theory and involves four stages: (i) identifying, (ii) planning, (iii) implementing, and (iv) evaluating. In each stage of the solution process, the teacher has to do explicit modeling through think-aloud. Five kinds of metacognitive questions: comprehension question, connection question, strategic question, checking question, and reflection question are used to demonstrate what is going on in the teacher’s head and how to monitor and control the thinking process while solving the problem.

(v) Consolidating in collaborative setting. In this stage, students are formed into heterogeneous learning group and they have to solve the problems by taking the role of thinker and listener. The thinkers have to explain their reasoning through verbalizing, while the listeners have to listen, record, ask questions, and make sure what the thinkers say. A set of metacognitive question cards are delivered to each group to help students to be aware of and monitor the problem solving phases.

(vi) Evaluating performance and transferring learning. In this stage, students have to solve the problem independently. They have to do think-aloud about all the steps to monitor and control their thinking process. Next, they have to do reflective writing.

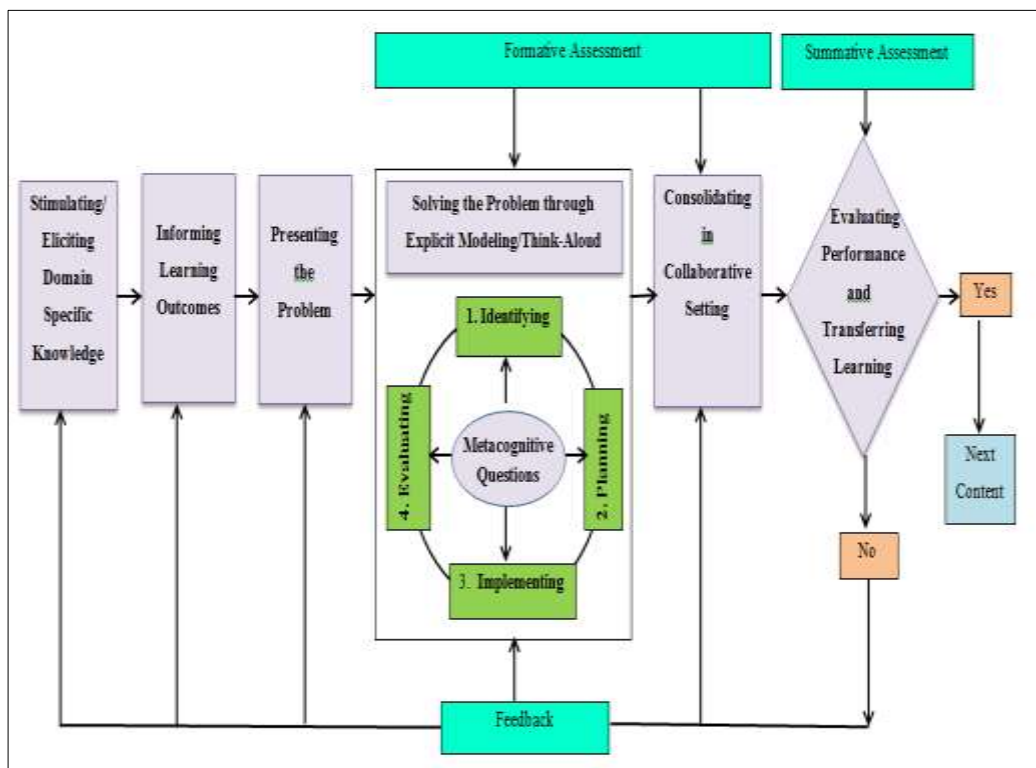


Figure 1 Proposed Metacognitive Teaching Model for Problem Solving in Middle School mathematics

Research Method

The explanatory sequential (QUAN \rightarrow qual), one of the basic mixed methods designs was adopted in this study.

Quantitative Research Method

Research design. The quasi-experimental research design was adopted for the quantitative study. The nonequivalent control group pretest/posttest design was adopted to explore the answer to the research question (1) and the single group pretest/posttest design was adopted to answer the research question (2).

Subject. Table 1 shows the sample size of the quantitative study.

Table 1 Population and Sample Size for Quantitative Study

No.	District	Township	School	Population	Sample	Group	No. of Student
1	East	Yankin	Practicing Middel School Yankin (S1)	67	62	EG	31
						CG	31
2	West	Mayangone	No. (5) BEHS, Mayangone (S2)	129	58	EG	29
						CG	29
3	South	Thanlin	No. (2) EEHS, Thanlyin (S3)	79	72	EG	36
						CG	36
4	North	Mingaladon	No. (4) BEMS, Mingaladon (S4)	144	66	EG	33
						CG	33
Total				419	258		258

Note. BEHS = Basic Education High School; BEMS = Basic Education Middle School;

EG = Experimental Group; CG = Control group.

Instruments. The main instruments to collect the data for the quantitative study are a pretest, a posttest, and a metacognitive skills inventory questionnaire.

Pretest. It was constructed based on Grade five mathematics. There are (30) multiple-choice items and the total score is (30) marks, and time allocation is (45) minutes.

Posttest. It was developed based on Grade six mathematics textbook volumes I and II. Test items were developed according to the five process skills of mathematics. The test consists of two sections. Section A consists of ten multiple-choice items and section B consists of four short answer questions (word problems). Time allocation is (1:00) hour, and the total score is (30) marks.

Metacognitive skills inventory questionnaire. The questionnaire is five points Likert scale. Items are categorized into four subscales: (i) identifying, (ii) planning, (iii) monitoring, and (iv) evaluation. There are eight items for each subscale, and a total of (32) items are included in the questionnaire.

Qualitative Research Method

Research design. The instrumental case study (observation) was adopted to get the evidence to support the quantitative results.

Subject. In each school, depending on the result of the pretest scores, experimental group students were assigned to groups A, B, and C. In each group, a student was elicited by using random purposive sampling. Thus, a total of (12) students participated in the observation.

Instrument. The main instrument for the qualitative study is the observation checklist.

An observation checklist. It was developed by adapting the framework for cognitive and metacognitive problem solving behavior (Artzt & Armour-Thoms, 1992). It includes (30) observed behaviors: (3) cognitive and (22) metacognitive behaviors. The presence or absence of each behavior is recorded once at every (3) minute intervals and the total length of time for observation is (30) minutes.

Teaching-Learning Materials

A total of (38) lesson plans were developed based on the four chapters from Grade six mathematics volume I and two chapters from mathematics volume II.

Study Procedure

This study was started in the first week of November 2021 and ended in the second week of January 2022. The duration of the study was taken about eleven weeks.

The pilot study. The pilot study was conducted with (25) Grade six students from No. (2) Basic Education Middle School, Yankin. During the pilot study, students were taught problem solving through the stages of the proposed model. They were administered the pretest, posttest, and metacognitive skills inventory questionnaire. The internal consistency of the pretest was 0.737, the posttest was 0.752, and the metacognitive skills inventory questionnaire was 0.894 respectively.

The main study. The main study was started in the first week of November 2021 and ended in the second week of January 2022.

Quantitative study. Before starting the treatment period, the two intact groups from each selected sample school were randomly assigned as experimental and control groups. Then, a pretest was administered to both groups. Next, the metacognitive skills inventory questionnaire was administered only to experimental groups. The duration of treatment was taken about eight weeks and a total of (30) hours of treatment periods were taken during eight weeks.

During the treatment period, both groups were taught the same lesson contents by the same mathematics teachers. The instructional difference between the two groups was the experimental groups were taught problem solving through the metacognitive teaching model and the control groups were taught problem solving through the formal problem solving instructional procedure. About three weeks before the end of the study period, experimental groups were administered the metacognitive skills inventory questionnaire. After the treatment period, the two intact groups were sat on a posttest.

Qualitative study. About two weeks before the end of the study period, observation was conducted. To observe and record the cognitive and metacognitive problem solving behaviors, each student was scheduled for an individual session in a quiet setting. They were facilitated to explore rich verbal data about reasoning during problem solving. During each of the three-minute intervals, the problem solving behaviors of each student was observed and video recorded.

Data analysis. The pretest and posttest data were analyzed through one-way ANCOVA, and students' responses to the metacognitive skills inventory questionnaire were analyzed through Wilcoxon Signed Rank Test. The data got from observation were analyzed through think-aloud protocol analysis.

Research Findings

Quantitative Research Findings

(i) Findings of Grade Six Students' Mathematics Basic Knowledge on Pretest

Table 2 shows the findings for mathematics basic knowledge of Grade six students.

Table 2 Results for Pretest Scores of Mathematics Basic Knowledge

School	Group	N	M	SD	MD	F	p
S1	Experimental	31	17.96	3.18	1.13	1.06	.306 (ns)
	Control	31	16.83	5.19			
S2	Experimental	29	16.48	4.09	2.83	7.53	.008**
	Control	29	13.65	3.74			
S3	Experimental	36	16.30	3.35	3.47	19.61	.000***
	Control	36	12.83	3.29			
S4	Experimental	33	16.69	3.61	2.00	4.94	.029*
	Control	33	14.69	3.65			

Note. ns = not significant, * $p < .05$, ** $p < .01$, *** $p < .001$.

The results in Table 2 show that there were significant differences in entry behavior between the two groups in S2, S3, and S4. There was no significant difference in entry behavior between the two groups in S1.

(ii) Findings of Grade Six Students' Mathematics Achievement on Posttest

Table 3 shows the findings for Grade six students' mathematics achievement on posttest.

Table 3 One-way ANCOVA and Descriptive Statistics Results for Posttest Score

School	Tests of Between-Subjects Effects					Observed Mean		Adjusted Mean	
	Source	df	F	p	η^2	EG	CG	EG	CG
S1	Pretest	1	2.10	.152(ns)	.034	20.59	15.29	20.49	15.39
	Group	1	24.10	.000***	.300				
	Error	59							
S2	Pretest	1	1.04	.312(ns)	.019	19.50	13.84	19.36	14.01
	Group	1	28.80	.000***	.344				
	Error	55							
S3	Pretest	1	11.73	.001**	.145	17.43	12.29	16.85	12.86
	Group	1	31.22	.000***	.312				
	Error	69							
S4	Pretest	1	13.08	.001**	.172	20.50	13.66	20.08	14.08
	Group	1	49.36	.000***	.440				
	Error	63							

Note. ns = not significant, ** $p < .01$, *** $p < .001$

The results in Table 3 show that there were significant differences between the two groups in all selected schools. Although there were significant relationships between the pretest scores (covariate) and the posttest in S3 and S4, the adjusted mean scores support that the control groups' mean scores were lower than the experimental groups. Therefore, the proposed model has a better achievement in mathematics than the formal problem solving instruction.

(iii) Findings of Grade Six Students' Metacognitive Skills on Problem Solving

Table 4 describes the comparison of the descriptive statistic results for experimental group students' responses to each item in the metacognitive skills inventory questionnaire before and after the treatment. Total of (129) students responded to the questionnaire.

Table 4 Descriptive Statistics Results for Experimental Group Students' Responses on Metacognitive Skills Inventory Questionnaire

Identifying		Before		After	
No.	Statement	M	SD	M	SD
1	Read aloud the problem statements	3.14	1.06	4.28	0.84
2	Read the problem statements silently	2.92	1.10	4.11	0.88
3	Underline the key words	2.77	0.99	4.02	0.84
4	Explore the kind of the problem	3.42	1.09	4.13	0.85
5	Think in advance the problem difficulty	3.48	1.25	3.90	0.89
6	Decompose the problem statements	2.48	1.00	3.85	0.87
7	Notice the presence or absence of information	3.06	1.00	3.87	0.82
8	Differentiate the relevant and irrelevant information	2.66	0.85	3.66	0.78
Planning		Before		After	
No.	Statement	M	SD	M	SD
9	Take time to design an action plan	3.31	1.19	3.98	0.85
10	Collect the relevant materials and equipment	2.86	1.06	3.82	0.83
11	Analyze the similarities and differences	3.53	1.12	4.33	0.85
12	Reflect on concepts, theorems, and laws	3.08	1.06	3.97	0.78
13	Take time to select the relevant strategy	3.14	1.13	3.82	0.87
14	Arrange the sequence of steps	2.62	1.03	3.80	0.85
15	Explore the solution of the problem	3.04	0.99	3.88	0.87
16	Check the accuracy of the planning steps	2.89	0.88	4.10	0.80
Monitoring		Before		After	
No.	Statement	M	SD	M	SD
17	Engage coherent and well-structured calculation	3.82	1.99	4.41	0.82
18	Keep track of what is going on	2.73	0.95	3.88	0.82
19	Make sure the operations in each step	3.63	1.14	4.31	0.78
20	Examine if any computation step are left or not	3.47	1.09	4.13	0.76

21	Monitor the ongoing problem solving process	3.17	1.14	3.87	0.84
22	Change the strategy if it does not work out	2.82	1.31	3.86	0.83
23	Work slowly in difficult numerical calculations	3.48	1.14	4.11	0.88
24	Aware of the mistakes while solving the problem	3.23	1.02	4.16	0.81
Evaluating		Before		After	
No.	Statement	M	SD	M	SD
25	Check the answer and the units	3.66	1.22	4.46	0.77
26	Check the numbers copied from the given problem	3.35	1.09	4.37	0.74
27	Look back the accuracy of the computation steps	3.06	1.01	4.20	0.83
28	Reflect on how the solution was done	2.95	1.12	3.86	0.76
29	Find alternate ways to get the solution	2.30	1.05	3.48	0.75
30	Re-examine the answer and nature of problem	2.84	0.88	4.02	0.88
31	Reflect on what went well and what did not go well	2.76	0.90	3.93	0.76
32	Decide how to change weak points	2.88	0.93	4.22	0.77

According to the results of the descriptive statistics described in Table 4, it was found that the mean scores of each item before the treatment were increased to after the treatment.

Table 5 describes the findings of the experimental group students' responses to the metacognitive skills inventory questionnaire before and after the treatment.

Table 5 Wilcoxon Signed Rank Test Results for Experimental Group Students' Responses on Metacognitive Skills Inventory Questionnaire

School	Pair	<i>N</i>	<i>M</i>	<i>SD</i>	<i>Md</i>	<i>z</i>	<i>P</i>	<i>r</i>
S1	Before	31	93.67	12.96	96	-4.86	.000***	.87
	After	31	121.16	10.23	121			
S2	Before	29	93.82	18.42	89	-4.71	.000***	.87
	After	29	135.55	13.46	137			
S3	Before	36	108.86	13.18	108	-5.23	.000***	.87
	After	36	129.19	13.95	132			
S4	Before	33	96.72	13.12	97	-5.01	.000***	.87
	After	33	130.06	14.73	128			

Note. *** $p < .001$.

The results in Table 5 point out that there were significant differences in the experimental group students' responses to the metacognitive skills inventory questionnaire before and after the treatment.

Qualitative Research Findings

(i) Findings of Observation on Problem Solving Behaviors of Student

Table 6 shows the problem solving behaviors of (12) students during an individual session of (30) minutes of observation.

Table 6 The Overall Structure of the Problem Solving Behaviors of Students

Reading the Problem													
No.	Point to Observe	S1	S2	S3	S4	S1	S2	S3	S4	S1	S2	S3	S4
		A1	A2	A3	A4	B1	B2	B3	B4	C1	C2	C3	C4
		%	%	%	%	%	%	%	%	%	%	%	%
1	Read Aloud (C)	20	60	20	40	20	30	60	40	30	20	30	30
2	Read Silent (C)	20	20	20	40	20	40	10	20	20	40	30	20
3	Underline (C)	10	20	20	10	20	20	10	20	20	10	10	20
Understanding the Problem													
4	Explore Type (M)	10	10	10	20	20	-	10	10	10	-	10	30
5	Paraphrase (M)	20	20	30	20	20	20	20	10	20	20	20	20
6	Drawing (M)	20	20	20	20	20	20	20	20	10	20	10	20
Analyzing the Problem													
7	Write (M)	30	40	20	20	20	20	30	30	10	30	30	20
8	Check Given (M)	-	20	20	10	-	10	10	20	-	10	20	-
9	Explore Similar (M)	10	20	30	20	30	30	-	30	20	-	10	20
10	Analyze (M)	10	20	20	20	20	-	-	10	10	-	20	20
Planning													
11	Explore Concept (M)	20	20	20	30	40	30	10	30	10	10	20	20
12	Select Strategy (M)	30	20	20	20	20	10	20	30	-	20	10	10
13	Collect Tools (M)	-	10	10	20	10	10	30	20	20	20	10	-
14	Arrange Steps (M)	50	20	-	10	20	10	-	20	-	10	-	-
15	Draft Calculation (M)	40	40	20	20	20	20	30	30	30	30	30	30
16	Check Plan (M)	-	-	-	-	-	-	-	-	-	-	-	-
Implementing													
17	Do Calculation (C)	20	40	30	20	40	30	30	30	30	30	50	40
18	Slow and Steady (M)	30	50	60	30	60	50	30	40	60	50	50	20
19	Verbalize Operations (M)	30	40	10	20	-	30	40	40	10	20	-	20
20	Check Symbols (M)	-	-	-	-	-	-	-	-	-	-	-	-
21	Check Steps (M)	10	30	20	30	10	40	20	20	-	10	30	20
22	Aware Mistake (M)	10	-	10	20	20	10	10	30	10	20	10	20
23	Revise (M)	10	-	10	20	20	10	20	10	10	20	20	20
24	Change Strategy (M)	-	-	10	-	-	-	10	-	10	-	-	10
Verifying													
25	Write Answer (C)	20	30	20	20	20	30	30	30	30	30	40	20
26	Check Answer & Unit (C)	20	20	20	20	20	20	20	10	10	20	20	20
27	Check Number (C)	-	20	20	10	10	10	10	10	20	10	20	20

28	Check Process (C)	20	20	20	20	20	10	20	10	20	10	20	10
29	Reexamine Answer (M)	20	20	-	-	20	10	20	-	30	10	-	-
30	Explore Strategies (M)	-	-	-	-	-	-	-	-	-	-	-	-

Note. C = Cognitive behavior, M = Metacognitive Behavior; A1, A2, A3, and A4 = Group A students; B1, B2, B3, and B4 = Group B students; C1, C2, C3, and C4 = Group C students

(ii) Findings on Cognitive and Metacognitive Problem Solving Behaviors of Students

Table 7 describes the summary of cognitive and metacognitive problem solving behaviors exhibited by (12) students during an individual session of (30) minutes of observation.

Table 7 Summary of Cognitive and Metacognitive Problem Solving Behaviors

No.	Student	Cognitive	(%)	Metacognitive	(%)	Total	(%)
1	A1	7/8	88	16/22	73	23/30	77
2	A2	8/8	100	16/22	73	24/30	80
3	A3	8/8	100	17/22	77	25/30	83
4	A4	8/8	100	17/22	77	25/30	83
Total/Average		31/32	97	66/88	75	97/120	81
No.	Student	Cognitive	(%)	Metacognitive	(%)	Total	(%)
5	B1	8/8	100	16/22	73	24/30	80
6	B2	8/8	100	16/22	73	24/30	80
7	B3	8/8	100	16/22	73	24/30	80
8	B4	8/8	100	17/22	77	25/30	83
Total/Average		32/32	100	65/88	74	97/120	81
No.	Student	Cognitive	(%)	Metacognitive	(%)	Total	(%)
9	C1	8/8	100	15/22	68	23/30	77
10	C2	8/8	100	15/22	68	23/30	77
11	C3	8/8	100	15/22	68	23/30	77
12	C4	8/8	100	15/22	68	23/30	77
Total/Average		32/32	100	60/88	68	92/120	77

(iii) Findings on Think-Aloud Protocol Analysis

The video recordings of problem solving behaviors of (12) students during individual sessions of (30) minutes observations were analyzed through think-aloud protocol analysis and the summary of the findings is presented as follows. According to the think-aloud protocol analysis, students exhibited:

- (100%) of behaviors under the category for reading the problem;
- (34) out of (36), (94%) of behaviors under the category for understanding the problem;
- (39) out of (48), (81%) of behaviors under the category for analyzing the problem;
- (52) out of (72), (72%) of behaviors under this category for planning;
- (71) out of (96), (74%) of behaviors under the category for implementing the problem;

- (54) out of (72), (75%) of behaviors under the category for verifying.
- Overall, (4) out of (22), (18.18%) metacognitive behaviors: check plan, check symbols, change strategy, and explore strategies were not observed apparently.

Discussion

According to one-way ANCOVA results on posttest scores, there were significant differences in the mathematics achievement on posttest between the two groups in each selected school. In addition, Wilcoxon Signed Rank Test results revealed that there were significant differences in the metacognitive skills before and after the treatment in each selected school. The results support the finding of Mullick-Martinez (2020) who found that students who were taught problem solving through metacognitive strategies have improvement both in mathematics achievement and metacognitive awareness. In addition, these findings are consistent with Teong (2000) who revealed that metacognitive training results in a greater improvement in mathematical word problem solving of the experimental group than that of the control group. These findings advocate Kendir and Sahin (2013) who found metacognitive strategies result in a significant difference in the metacognitive skills of students.

Almost all students in each group exhibited both cognitive and metacognitive behaviors in each category. Exceptionally, (4) out of (30) behaviors: check plan, check symbols, change strategy, and explore strategies were not observed apparently in most students. This finding is consistent with Ericsson and Simon (1993) who reminded that thoughts in normal form can proceed much more rapidly than speech. When a series of thoughts occurs rapidly, it is impossible for an individual to directly verbalize each and every thought in that series and these are vocalized as inner speech. Thus, it can be interpreted that they could perform these behaviors rapidly and some of their verbalizations seem to correspond to merely vocalizing as inner speech.

Suggestions

In mathematics, problem solving is the heart of the subject and it helps students to tackle the problems in their lives with confidence. Mathematics teachers should be familiar with metacognitive strategies and should train their students to develop metacognitive skills: identifying, planning, monitoring, and evaluating through teaching problem solving. In mathematics, the heuristic strategies support to the development of metacognition (Biggs & Telfer, 1987). Heuristics are often discussed in mathematics education at the secondary level, rather than at the primary level (MoE, 2019). Therefore, mathematics teachers at the middle school level should be familiar with heuristic strategies. Students should be engaged to assess reflective writings through self-assessment and peer-assessment. Metacognition could develop through social interaction. Thus, teachers should create different interaction patterns while teaching problem solving. In the present study, randomly assigned to the individual student to experimental and control groups was difficult in real context and a quasi-experimental design was adopted. Thus, a true experimental study should be conducted to generalize the present findings. More research studies are also needed to study the development of metacognitive skills across different age groups and/or the correlation between metacognitive skills and achievement.

Conclusion

According to the findings, the proposed metacognitive teaching model has supported the students' achievement in mathematics, and has provided the enhancement of the metacognitive skills of students concerning problem solving. Therefore, it can be concluded that this study could provide a reasonable solution for solving the underachievement problem of students in mathematics due to the lack of control and monitoring of what they are doing. Hopefully, the present study could contribute to some extent to the improvement of teaching mathematical problem solving at the middle school level in Myanmar.

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